Comment on "Multiple scattering: The key to unravel the subwavelength world from the far-field pattern of a scattered wave"

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Contrary to the main conclusion of Simonetti [Phys. Rev. E **73**, 036619 (2006)], we maintain that multiple scattering (MS) is not the "key" for subwavelength detection. Indeed, even with no MS between subwavelength structures, subwavelength detection is still possible. Our statement is numerically confirmed. A simple mathematical argument explains this result. From our point of view, the incorrect conclusion of Simonetti comes from a misinterpretation of the Picard's theorem.

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To begin with, we have performed a simple numerical computation corresponding to the experimental configuration presented in Ref. [1]. Two pointlike isotropic scatterers are considered. The $\lambda/3$ distance between them is smaller than resolution limit dictated by the Rayleigh criterion. For two isotropic scatterers, the backscattering T matrix writes

$$T_\infty = \begin{pmatrix}|e_1\rangle & |e_2\rangle\end{pmatrix}\begin{pmatrix}S_{11} & S_{21}\\ S_{12} & S_{22}\end{pmatrix}\begin{pmatrix}\langle e_1|\\ \langle e_2|\end{pmatrix}, \qquad (1)$$

where $|e_1\rangle$ (respectively $|e_2\rangle$) is given by the Green's function between scatterer 1 (respectively 2) and the array. S_{11}, S_{12}, S_{21}, and S_{22} are four scattering coefficients of the $\mathbf{S}$ matrix. Basically, S_{12} represents the "multiple" (at least two) scattering. Hence when $S_{12}=0$, there is no interaction between scatterers. In Fig. 1 are plotted the pseudospectrum estimators $P(\mathbf{z})$ [Eq. (35) in [1]] when $S_{12}=0$ and $S_{12}=\exp(-i2\pi\delta/\lambda)$. Length δ is the distance between the two scatterers. In both cases $S_{11}=S_{22}=1$. For comparison with the Rayleigh criterium, the conventional beam forming is also plotted. It clearly appears that even with no scattering between the two scatterers, the pseudospectrum estimator resolution is much better than $\lambda/3$. Consequently, in that case, the multiple scattering (MS) does not result in better subwavelength imaging. A simple mathematical analysis sustains this observation. Due to Eq. (1), the T-matrix operator is a linear projector on the two-dimensional subspace Γ generated by the vectors $|e_1\rangle$ and $|e_2\rangle$. Note that Γ does not depend on the $\mathbf{S}$ matrix. The two first singular vectors $|v_n\rangle$ of T_∞ associated to the two significant singular values form an orthogonal basis of Γ. Consequently, the pseudospectral estimator $P(\mathbf{z})$ [or $E(\mathbf{z})$] goes toward infinity when $|g_z\rangle$ belongs to Γ (i.e., $|g_z\rangle=|e_1\rangle$ or $|g_z\rangle=|e_2\rangle$). Indeed, in such a case, $|\langle g_z|v_n\rangle|$ equals 0 for $n>2$ [See Eq. (35) in [1]]. Clearly, this analysis can be generalized without difficulty to M pointlike scatterers. This mathematical approach explains why linear sampling method or factorization method consists of nonlinear super-resolving detection techniques that do not depend on MS (as long as the subspace Γ does not depend on MS).

Following the author arguments, the super-resolution of $P(\mathbf{z})$ is due to the Picard's theorem [see Eq. (34)]. However, the expression of the Picard's theorem, at least as it is explained in the paper, remains exactly the same whether the MS is considered or not in Eq. (24). Hence $P(\mathbf{z})>0$, when $z\in D$ regardless of MS, which is in contradiction with the last sentence of the paragraph following Eq. (35).

In other words, the free space Green's function $|g_z\rangle$ [see Eq. (30)] that does not include MS can always be used as a testing function to localize the last scattering positions. Without MS, $P(\mathbf{z})$ is larger than zero and proportional to the local scattering coefficient when $\mathbf{z}\in D$. In presence of MS, $P(\mathbf{z})$ is still larger than 0 when $\mathbf{z}\in D$, but now $P(\mathbf{z})$ deviates from the local scattering coefficient because of the MS.

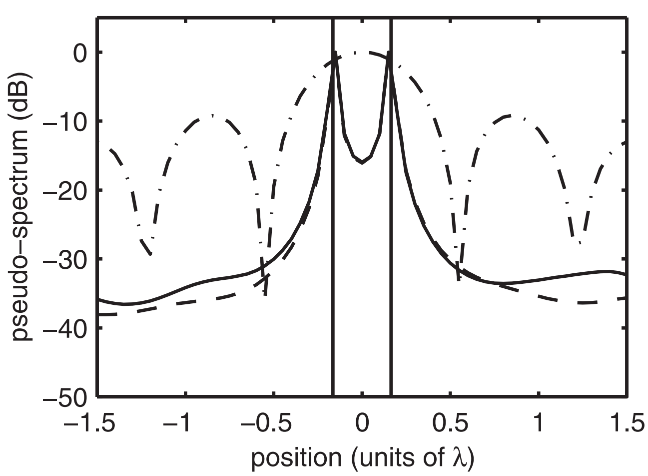

FIG. 1. Cross section of super-resolved image at 42 kHz: (solid line) no interaction between scatterers; (dashed line) strong interaction. (dashed-dot line): beam forming. The two vertical bars indicate the positions of the two scatterers.

Hence, MS should be taken into account for inversion problems. But here we are talking about detection. In such a case, the MS may either increases or decreases an echo from a "hidden" subwavelength structure and therefore helps or degrades the detection in a noisy environment.

In conclusion, the arguments given in Ref. [1] do not prove that "multiple scattering is the *key* to unravel subwavelength world."

[1] F. Simonetti, Phys. Rev. E **73**, 036619 (2006).